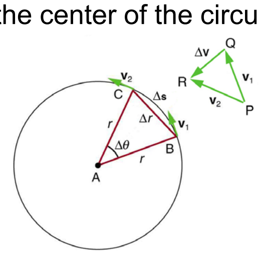
Circular Motion

Uniform Circular Motion

- An object that moves in a circle at constant tangential (linear) speed v, is said to experience **uniform circular motion**.
- The speed may be constant but the direction is changing.
- This means that the object is accelerating.
- The acceleration is in the direction of the change in velocity.

- The direction of the acceleration is towards the center of the circular path.



- This acceleration is called **centripetal acceleration** (towards the center or center seeking).

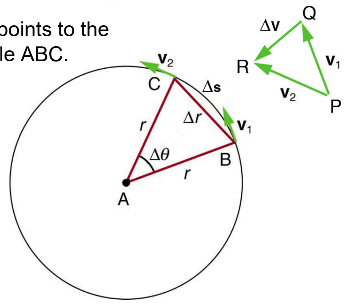
Deriving an Equation for Centripetal Acceleration

- During Δt the object moves from B to C
  - Connecting these points to the center gives triangle ABC.

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = v_2 - v_1$$

- The vector subtraction gives triangle PQR.
  - $v_1 = v_2 = v$



- The two triangles are similar triangles, therefore

$$\frac{\Delta v}{v} = \frac{\Delta r}{r}$$

- Solving for Δv

$$\Delta v = \frac{v \Delta r}{r}$$

- Divide both sides by Δt

$$\frac{\Delta v}{\Delta t} = \frac{v \Delta r}{r \Delta t}$$

$$\frac{\Delta v}{\Delta t} = a \qquad \frac{\Delta r}{\Delta t} = v$$

$$\boxed{a_c = \frac{v^2}{r}}$$

- It is also useful to express centripetal acceleration in terms of angular velocity, period and frequency.
  - The period, T, is the time required for one rotation.
  - The frequency, f, is the number of rotations per second.
  - Period and frequency are the inverses of each other.

$$\boxed{T = \frac{1}{f}}$$

$$a_c = \frac{v^2}{r} \qquad v = \frac{d}{t} \qquad d = 2\pi r$$

$$t = T$$

$$v = \frac{2\pi r}{T}$$

$$a_c = \frac{4\pi^2 r^2}{rT} = \frac{4\pi^2 r}{T}$$

Substituting $f = \frac{1}{T}$ $\qquad a_c = 4\pi^2 r f^2$

Example 1

- A car drives around a curve of radius 500.0 m at a speed of 25 m/s. Calculate the centripetal acceleration.

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{(25)^2}{500}$$

$$a_c = 1.25 \text{ m/s}^2$$

Example 2

- A 150 g ball at the end of a string is revolving in a horizontal circle of radius 0.60 m. The ball makes 2 revolutions in one second. Calculate the centripetal acceleration of the ball.

$$a_c = \frac{v^2}{r} = r\omega^2 = 4\pi^2 r f^2$$

$$a_c = 4\pi^2(0.6)(2)^2$$

$$a_c = 95\ \mathrm{m/s^2}$$

Centripetal Force

- Any force or combination of forces can cause a centripetal or radial acceleration.
- Any net force causing uniform circular motion is called a centripetal force.
- The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration.

- According to Newton's second law of motion, $F_{net} = ma$
- For uniform circular motion, the acceleration is the centripetal acceleration ($a = a_c$).
- Thus, the magnitude of centripetal force is

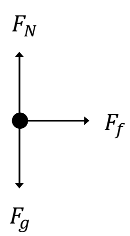
$$F_c = ma_c$$

or

$$F_c = m\frac{v^2}{r} = mr\omega^2$$

Example 1

- A 1200 kg car travels around a flat curve with a radius of 500.0 m at 25.0 m/s. Calculate the minimum static coefficient of friction between the tires and the road required to keep the car from slipping.



x

$$F_{net} = ma$$

Since the car is moving in a circle

$$F_{net} = F_c = ma_c$$

$$F_f = ma_c$$

y

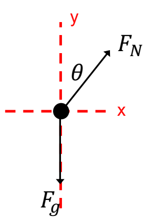
$$F_N = F_g = mg$$

$$\mu = \frac{F_f}{F_N} = \frac{ma_c}{mg} = \frac{mv^2}{mgr} = \frac{v^2}{gr}$$

$$\mu = \frac{(25)^2}{(9.8)(500)} = 0.13$$

Example 2

- A 1200 kg car travels around a banked, frictionless curve with a radius of 500.0 m at 25.0 m/s. Calculate the angle of the banked curve required to keep the car on the road.



x

$F = F_c = ma_c$ (Since the car is moving in a circle)

$$F_N \sin\theta = ma_c = \frac{mv^2}{r}$$

y

$F = ma$

$$F_N \cos\theta - mg = ma = 0$$

$$F_N \sin\theta = \frac{mv^2}{r}$$

$$\frac{mg\sin\theta}{\cos\theta} = \frac{mv^2}{r}$$

$$\tan\theta = \frac{v^2}{gr}$$

$$\theta = \tan^{-1}\left(\frac{25^2}{(9.8)(500)}\right) = 7.5°$$

$$F_N \cos\theta - mg = 0$$

$$F_N \cos\theta = mg$$

$$F_N = \frac{mg}{\cos\theta}$$